PART II
DESIGN OF THE FUTURE

Chapter 5
PRINCIPLES OF INTELLIGENT OPTIMAL DESIGN OF MATERIALS AND STRUCTURES

5.1 PRINCIPLES

5.1.1 Elementary Example

We are going to optimally design an air turbine disk, which is required to have

i) a long fatigue life, which implies that the stresses in any point have to be smaller than the endurance limit of the material,

Max $\left\{\sqrt{J_2}\right\} \le \sigma_D$, where J_2 is the second invariant of the stress tensor.

ii) an improved performance, by increasing the volume of the air being turned around.

But an improvement of the performance implies an increase of the mass, and then a more powerful engine. For a given speed, the external radius, R_E, of the disk is defined by some thermodynamic conditions.

In this design problem, there are only two design parameters to deal with:

h, the height and R_I , the internal radius of the disk.

A very practical solution was given to this problem, as follows:

1. After making several elastic analyses of the disk (with a finite-element code), it was noted that an heuristic rule could be assumed
   Max $\left\{\sqrt{J_2}\right\} \cong A_0 + A_1 h + A_2 R_I$.
   The coefficients in this linear relation between Max $\left\{\sqrt{J_2}\right\}$, h and R_I were identified after 3 computations. Other ones have shown the validity of the relations.
2. The volume of the disk, $V(h, R_I)$, then its weight is an analytical linear function of h and R_I .
   By introducing this linked relation into Max $\left\{\sqrt{J_2}\right\}$,
3. we get Max $\left\{\sqrt{J_2}\right\} = A_0 + A_1 h + A_2 R_I = \sigma_D$,
4. the volume can then be expressed as a function of h only, $V = V_0 + V_1 h + V_2 h^2 + V_3 h^3$.
5. The solution corresponds to $dV/dh = 0$ (here a free extremum of the function).
6. We have to solve a simple quadratic equation for h_{opt} and we then obatin R_{Iopt}.

This problem underlines the sucessive steps that we have to take. The general case, however, is more difficult, because we will have many design parameters, and a manual analysis for searching some relationships is almost impossible.

5.1.2 General Principle

The approach is based on automatic learning and optimization techniques where the whole knowledge of experts and experimental results are mixed. It is then needed to take the following steps:

i) Build a DATA BASE, i.e., obtain some experimental, real or simulated results, in which the EXPERTS of each particular problem identify all the variables or descriptors which may take part.

This is, at first, done with some PRIMITIVE descriptors x which are usually available in a limited number.

Then, the experts transform the data by introducing INTELLIGENT descriptors XX which may generally constitute a larger number. These descriptors represent the experts' actual whole knowledge and all their beautiful (but often insufficient) theories!

The descriptors may be numbers, boolean or alphanumeric, name of files to give access to data bases, or treatments of curves, signals or images.

The results or conclusions may be classes (good, not good, ...) or numbers (Young modulus, cost, weight, life time...).

Usually, the data base may contain $\cong$ 20 to 50 examples with 10 to 1000 descriptors and 1 to 20 conclusions.

ii) Generate the RULES with any automatic learning tool. The intelligent descriptors help these learning algorithms. Each conclusion is explained as function or set of rules of some among the input intelligent descriptors with a known reliability or accuracy. If this reliability is too low, it implies that wether there is no enough data or there are bad, missing descriptors or the problem was not well described.

iii) Optimize at two levels (inverse problems)

- Considering the intelligent descriptors as independent; it is possible to get the OPTIMAL SOLUTION satisfying the special required properties and allowing the DISCOVERY OF NEW MECHANISMS,
- Considering, the intelligent descriptors linked to primitive descriptors; it is possible to obtain the optimal solution which is technologically possible.

So, not only a practical optimal solution is obtained (since good interpolations and extrapolations for new cases are made) *but also the experts of the problem may learn the missing parts, may build models or theories based on the retained intelligent descriptors only, and guided by the shapes of the rules or relationships.*

Of course, general tools need to be defined to characterize the mechanical problems, and then to find the intelligent descriptors.

5.2 REPRESENTATION OF VARIOUS QUANTITIES

5.2.1 Objects to be described

Different representations of various quantities will be useful in the design of structures.

Domain Ω in R^n: Its boundary $\partial\Omega$ is known and may contain other information such that $\partial\Omega$ is splitted into two parts $\partial_u\Omega$ and $\partial_F\Omega$

Scalar field on Ω :Most of quantities to describe are set as $\varphi = f(x, y)$ or $\varphi = f(x, y, z)$ and define surfaces on a contour or hypersurfaces on a volume.

Tensorial field φ in $\Omega \subset R^n$: the simplest way to describe such field is to consider some equivalent scalar fields.

These fields may change with the time $\varphi = f(x)$ is defined on Ω in the time interval (t_1, t_2).The time may have a real physical meaning. In such a case, the dimension of the space may be increased of one, $\varphi\ (x, t)$ is defined in $\Omega\times(t_1, t_2)$, domain in R^{n+1}. or it is possible more difficultly to consider some bounds of φ. during the time The time may be also a simple evolution parameter, to give the order of the process Some quantities such as the mean , the maximum or the minimum values of φ on the time interval may be kept.

5.2.2 Characteristics of an Object

Three families of descriptors may be introduced to characterize an object.

Size:

The absolute value of its surface or its volume may sometimes be important.

For a scalar field $\varphi\ (x, y)$, on two-dimensional Ω_0 with the surface S_0, the volume delimited under the surface φ is equal to $V_{total} = \int_{\Omega_0} \varphi\ (x, y)\ dxdy.$

But more often, only the mean value $\varphi_{mean} = V_{total}/S_0$ will be taken as the size parameter

Position:

This description, if it has a meaning for the problem, needs to be defined in a fixed frame for all observers. It is natural to take the frame along the principal directions of inertia.We may have to keep the coordinates of the center of inertia, the principal axis of inertia, .. .

Shape:

Several criteria based on automatic shape representation may be found. Some methods work on the boundary of the object, other ones use transfomations to deduce scalar quantities.

Usually, a mathematical or morphological description such as the grouping technique can be used.

Signals: they can be characterized by their minimum, maximum, mean value, the number of passages through zero, their spectrum, but also by the rain-flow method.

Images: after scanning and digitalization, it is possible to extract contours and gray level variation, and/or introduce various filtering techniques for grain texture and distribution.

Geometrical invariants:

Very often we define, similar to statistics, for a discrete variable X with n values $X_1, X_2, X_3, ..., X_n$, or for a continuous variable $X(x)$,

its mean value $X_{mean} = \sum \frac{X_i}{n}$

and the centered moments of order p $m_p = \frac{1}{n} \sum_{i=1}^{n} (X_i - X_{mean})^p$, which may be rendered non-dimensional by introducing $a_p = (m_p)/m_2^{p/2}$, where m_2 is the moment of order p = 2 or the variance.

The 3rd order moment is the skewness and the 4 th order moment is the kurtosis (which, for a Gauss distribution, takes the value 3).

Such a representation is possible for any multidimensional variable and for any domain Ω_0 with the surface S_0 , for which the shape tensors are $T(m,n)$ of order m on an n-dimension domain, and has the generic term

$t_{i_1,i_2,..,i_m} = \int_{\Omega_0} x_{i_1} x_{i_2} ... x_{i_m} \, d\Omega \;\; with \; i_1, i_2, ..., i_m = 1, ..., n.$

Classical invariants are then computed.

In a plane domain, the tensor $T_{2,2}$ has four terms

$t_{11} = \int_{\Omega_0} x^2 \, d\Omega = I_{XX}$;

$t_{12} = t_{21} = \int_{\Omega_0} xy \, d\Omega$;

$t_{22} = \int_{\Omega_0} y^2 \, d\Omega$.

(with the same invariants that the classical moment of inertia)

The use of -:

-gyration radius $r_g^2 = \frac{(I_{XX}+I_{YY})}{S_0} = \frac{1}{S_0} \int_{\Omega_0} (x^2 + y^2) \, d\Omega$

- reference radius $r_r = \sqrt{\frac{S_0}{\pi}}$

allows to reach adimensional descriptors (terms in x_i are divided by r_g , ,the other invariants are divided by factors $r_g^{\alpha} \, r_r^{\beta}$ so the second invariant $I_{XX} + I_{YY}$ of the tensor of order 2 is replaced by the ratio $\frac{S_0}{r_g^2}$...).

The reduced invariants will have the expressions:

- for the 2nd order, the two invariants
  - $a_{21} = \frac{S_0}{r_g^2}$
  - $a_{22} = \frac{(I_{XX}+I_{YY} - I_{XY}^2)}{S_0^2 \, r_g^4}$
- for the 3rd order tensor

$$\begin{cases} t_{111} = \int_{\Omega} x^3 dxdy \\ t_{112} = t_{121} = t_{211} = \int_{\Omega} x^2 y dxdy \\ t_{122} = t_{211} = t_{221} = \int_{\Omega} xy^2 dxdy \\ t_{222} = \int_{\Omega} y^3 dxdy \end{cases}$$

  the two invariants

  - $a_{31} = \frac{t_{111}^2 + 3*t_{112}^2 + 3*t_{122}^2 + t_{222}^2}{S_0^2 * r_g^6}$
  - $a_{32} = \frac{t_{112}*(t_{222} - t_{112}) + t_{122}*(t_{111} - t_{122})}{S_0^2 * r_g^6}$
- for the 4th order tensor,

$$\begin{cases} t_{1111} = \int_\Omega x^4 dxdy \\ t_{1112} = t_{1121} = t_{2111} = t_{1211} = \int_\Omega x^3 ydxdy \\ t_{1122} = t_{1221} = t_{2211} = t_{2121} = t_{1212} = \int_\Omega x^2y^2 dxdy \\ t_{1222} = t_{2221} = t_{2212} = t_{2122} = \int_\Omega x^3 ydxdy \\ t_{2222} = \int_\Omega y^4 dxdy \end{cases}$$

the 3 invariants

- $a_{41} = \frac{t_{1111} + 2*t_{1122} + t_{2222}}{S_0 * r_g^4}$
- $a_{42} = \frac{t_{1111}^2 + 6*t_{1122}^2 + 4t_{1112}^2 + 4t_{1222}^2 + t_{2222}^2}{S_0^2 * r_g^8}$
- $a_{43} = \frac{(t_{1111} + t_{1122})^2 + 2(t_{1112} + t_{1222})^2 + (t_{1122} + t_{2222})^2}{S_0^2 * r_g^8}$

Other terms can be created such as the perimetric ratio
$r_p = \frac{perimeter\ of\ \Omega_0}{perimeter\_of\ a\_circle\ with\ same\ area\_as \Omega_0}$

5.2.3 Scalar Field φ on a Domain

The extrema φ_{min} and φ_{max} are linked to the shape of the function φ in Ω_0(shape descriptors sensitive to the mesh used for the domain are to be handled with care) but the mean φ_{mean} in Ω_0 is a size descriptor (more independent from the mesh).

It is possible to cut the surface delimited by φ in slices (φ_i ,φ_{i+1}) in the interval (φ_{min} , φ_{max}) and to consider for each of these slices the area S_i of the parts $\Omega_i \subset \Omega_0$ for which M $\in \Omega_i \implies \varphi_i \leq \varphi$(M) $\leq \varphi_{i+1}$. The repartition may be defined by $\frac{S_i}{S_0}$.

A filling coefficient for the volume V_{total} delimited by the surface $\varphi = f(x,y)$ may be given with $\frac{(Vtotal)}{(\varphi_{max}.S_0)}$ which is always ≤ 1

- The volume defined by the difference relative to the mean value:
  $\frac{1}{V_{total}} \int_\Omega |\varphi(x,y) - \varphi_{mean}|\, dxdy.$

5.2.4 Elementary Shapes

Coding a plane surface

For any plane surface Ω defined in Ω_0, the parameters are (in a common frame of reference and with the same units system) with area S :

- size parameter

  $\frac{S}{S_0}$.

- shape parameters

  the perimetric ratio,
  $r_p = \frac{perimeter\ of\ \Omega_0}{perimeter\_of\ a\_circle\ with\ same\ area\_as \Omega_0}$

and the seven moments a_{ij} of order 2 to 4,
$a_{21} = \frac{S}{r_g^2}$, $a_{22} = \frac{I_{XX}\ I_{YY} - I_{XY}^2}{S^2.r_g^4}$.

- position parameters
  the coordinates of its inertia center G_Ω as the angle θ between the reference frame of Ω_0 and the principal axis of Ω give
  $\frac{x_{G_\Omega}}{r_g}$, $\frac{y_{G_\Omega}}{r_g}$, $\cos\theta$, $\sin\theta$.

Coding a volume

The volume included between the plane domain Ω_0 and any surface $\varphi = f(x, y)$ set on Ω_0 may be characterized by the parameters:

- Size parameters

  φ_{mean}

- Position parameters
  $\frac{x_{G_\Omega}}{r_g}$, $\frac{y_{G_\Omega}}{r_g}$, $\cos\theta$, $\sin\theta$ $.pp_4$
- Shape parameters

by the sub-volumes in number NSV which have a volume equal to k times the total volume(for example k = 0.5 , 0.6 ..0.9) or by NTR slices of $\varphi = f(x, y)$ by a horizontal plane at a given altitude (as in topographical cards).

Coding a surface

Characteristics of a Curve

Characteristics of a Signal

Characteristics of a Picture